

Comparison Study of Horizontal, Vertical and Combined Noise Image Deblurring Algorithm

¹Sunny, ²Kiran Jain

^{1,2}Doon Valley College, Karnal, Kurukshetra University, Hararyana, India

Abstract

An image is a representation of our visual perception. Images are integral part of our technology. But the major problem arises when the image is blue or contain the noise. The blurriness in the image is difficult to avoid and sometimes ruin the complete image. So in this paper we are going to discuss the techniques to deblur the image. We are going to propose the PDE based image deblurring model. On the basis of the PDE model, the comparison of 3 algorithms horizontal, vertical and combined deblurring is done in this paper.

Keywords

Discretization, Partial Differential Equation, Vertical Deblurring, Horizontal Deblurring, Matlab

I. Introduction

An ideal camera or recording device would record an image so that the intensity of a small piece (pixel) of the recorded image was directly proportional to the intensity of the corresponding section of the scene being recorded. The real cameras violate this model in two ways. First, the recorded intensity of a pixel is related to the intensity in a larger neighborhood of the corresponding section of the scene. This effect in visual images is called blurring. Second, the recorded intensities are contaminated by random noise. The Noise is unwanted or undesirable information that contaminates an image. Noise appears in images from a variety of sources. A blurred or degraded image can be approximately described by this equation $k = Hf + n$, where the k is the blurred image, the H is the distortion operator also called the point spread function (PSF) [5], f is the original true image, n is the additive noise, introduced during image acquisition, that corrupts the image.

Image restoration is the pre-processing method that targets to suppress degradation using knowledge about its nature. Restoration methods [3-4] attempts to recover an image that has been degraded using a priori knowledge of the degradation phenomenon. Hence, the restoration techniques are focussed towards modelling the degradation and applying the inverse process in order to recover the original image. The relative motion between the camera and the object may lead to blurring of image during its formation on the film of the camera. The various methods to estimate the degradation function for use in restoration are observation, experimentation and mathematical modelling. Here, PDE based mathematical modelling is proposed to model the degradation and recovery process.

Several restoration methods such as Weiner Filtering [2], Inverse Filtering, Constrained Least Squares, Lucy-Richardson [6-7] iteration have been proposed in literature that remove the motion blur either using Fourier Transformation in frequency domain or by using optimization techniques. The main difficulty with these methods is to estimate the deviation of the restored image from the original image at individual points that is due to the mechanism of these methods as processing in frequency domain. Another method, the travelling wave de-blurring method is a approach that works in spatial domain but the mathematical model discussed in this paper is not generalized and discretization issues and stability

criteria of differential equation has not been addressed. In fact, when the proposed differential equation is discretized using forward differencing scheme is unconditionally unstable which may not produce the desired results.

A generalized PDE [1] based image model is proposed to model the phenomenon of blurred image formation due to relative motion between camera and the object and further the recovery of original image in spatial domain. Lax scheme is used to discretize the resulting PDE which is mathematically stable and produces good result. Therefore, with the use of Lax method for discretizing the proposed PDE that was initially a flux conservative equation transforms to a ID flux conservative equation with an added diffusion term which is in the form of Navier-Stokes equation. This, additional diffusion term contributes towards further smoothing of image.

II. Proposed PDE based Image Deblurring Model

Let vector $\underline{X} \in R^n$, $f: R^n \rightarrow R$ and $\underline{X} = (x_1, x_2, x_3, \dots, x_n)$ and f is a function of $\underline{X}$. For 1D object $f(\underline{X}) = x$ and for 2D object i.e. images $f(\underline{X}) = (x, y)$. Let $\underline{V}$ represents the velocity vector of object and $\underline{V} = (v_1, v_2, \dots, v_n)$. If object is moving in horizontal direction only then velocity reads as $\underline{V} = v_x$ and if object is under motion in XY-space in both horizontal and vertical directions then velocity vector reads as $\underline{V} = (v_x, v_y)$. If n-dimensional object $f(\underline{X})$ keeps a linear uniform motion at a rate $\underline{V}$ in n-Dim space under the surveillance of a camera. The total exposure $g(\underline{X}, t)$ at any point of the recording medium (e.g., film) is obtained by integrating the instantaneous exposure over the time interval $0 \leq t \leq T$ during which camera shutter is open.

Observed object for duration T can be modeled as –

$$g(\underline{X}, t) = \int_0^T f(\underline{X} - \underline{V}t) dt \quad (1)$$

Differentiating Equation (1) w.r.t t following equation is obtained:

$$g_t(\underline{X}, t) = f(\underline{X} - \underline{V}t)$$

After multiplying equation (1) by $\nabla \underline{X}$ where

$$\nabla \underline{X} = \left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \dots + \frac{\partial}{\partial x_n} \right) \underline{X}, \text{ it reads}$$

$$\nabla g(\underline{X}, t) = \int_0^T \nabla f(\underline{X} - \underline{V}t) dt \quad (2)$$

Differentiating Equation (2) w.r.t t , RHS reads-

$$\nabla \frac{d}{dt} f(\underline{X} - \underline{V}t) = -(\underline{V} \cdot \nabla) f(\underline{X} - \underline{V}t)$$

$$\Rightarrow f(\underline{X} - \underline{V}t) - f(\underline{X}) = -(\underline{V} \cdot \nabla) \int_0^T f(\underline{X} - \underline{V}t) dt$$

$$\Rightarrow g_t(\underline{X}, t) - f(\underline{X}) = (\underline{V} \cdot \nabla) g(\underline{X}, t)$$

$$\Rightarrow f(\underline{X}) = g_t(\underline{X}, t) + (\underline{V} \cdot \nabla) g(\underline{X}, t)$$

Therefore, the generalized equation that recovers original image $f(\underline{X})$ from observed blurred image $g(\underline{X}, t)$ is given by:-

$$f(\underline{X}) = \frac{\partial g(\underline{X}, t)}{\partial t} + \sum_{k=1}^n V_k \frac{\partial g(\underline{X}, t)}{\partial X_k} \quad (3)$$

When object or signal is 2-D i.e. images and object $f(\underline{X}) = (x, y)$ is moving in XY-space with uniform velocity $\underline{V} = (v_x, v_y)$ with X-component v_x and y-component v_y and camera shutter is open for duration 0 to T. Then, the observed image $g(x, y, t)$ can be expressed as:-

$$u(x, y, t) = \int_0^T f(x - v_x t, y - v_y t) dt \quad (4)$$

Then original image $f(x, y)$ can be recovered from motion blurred observed image $g(x, y, t)$ as follows derived from equation (4)

$$f(x, y) = \left(\frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} \right) g(x, y, t) \quad (5)$$

$$f(x, y) = g_t + v_x g_x + v_y g_y$$

if $v_x = v_y = v$ then

$$f(x, y) = \left[\frac{\partial}{\partial t} + v \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) \right] g(x, y, t) \quad (6)$$

Hence, Image restoration model for motion blur removal reads-

$$f(x, y) = \left[\frac{\partial}{\partial t} + v \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) \right] g(x, y, t) \quad (7)$$

With $g(x, y, 0) = f(x, y) = 0$

For only uniform horizontal motion $V_y = 0$ and $V_x = v$.

III. Discretization, Mathematical Analysis and Implementation of the Proposed PDE

Let us consider the very simple case where the object is moving in horizontal direction with uniform velocity $v_x = v$ and $v_y = 0$ and camera is fixed and its shutter is open for time duration T. Also at $t=0$, $f(x, y)=0$.

Therefore under this condition, equation (7) reads

$$\frac{\partial g}{\partial t} = -v \frac{\partial g}{\partial x} \quad (8)$$

This is known as ID flux conservative equation or advective equation because the quantity g is transported by a fluid flow with velocity v and this concept can be used for modeling the information transport in image. Also analytically the general solution of this equation is a wave propagating in the positive x-direction is $g=f(x-vt)$ which models our image blurring process.

For discretizing equation (8) using Finite Time Centred Scheme (FTCS), consider equally spaced points along both the t-axes and x-axes and denote

$$x_j = x_0 + j\Delta x, \quad j = 0, 1, 2, \dots, m \quad \text{and}$$

$$t_n = t_0 + n\Delta t, \quad n = 0, 1, 2, \dots, n \quad \text{where } m \times n \text{ is the size of image.}$$

Let g_j^n denote $g(t_n, x_j)$. Then, the resulting finite difference approximation to equation (4.8) reads

$$\left(\frac{g_j^{n+1} - g_j^n}{\Delta t} \right) = -v \left(\frac{g_{j+1}^n - g_{j-1}^n}{2\Delta x} \right) \quad (9)$$

Which can be arranged to compute g_j^{n+1} .

This FTCS representation (9) is an explicit and single level scheme i.e. g_j^{n+1} for each j can be computed explicitly from the quantities that are already known. Being single level scheme, only values at time level n have to be stored to find values at time $n+1$. But, from the von Neumann stability analysis, the FTCS scheme is unconditionally unstable which may not produce good results. The instability problem can be addressed by a simple change by Lax method for discretizing the PDE of the proposed model.

Here, the term g_j^n in the time derivative term is replaced by its average

$$g_j^n = \frac{1}{2} (g_{j+1}^n + g_{j-1}^n)$$

and substituting this in equation (4.10), it reads

$$g_j^{n+1} = \frac{1}{2} (g_{j+1}^n + g_{j-1}^n) - \frac{v\Delta t}{2\Delta x} (g_{j+1}^n - g_{j-1}^n)$$

which after rearrangement reads

$$\frac{g_j^{n+1} - g_j^n}{\Delta t} = -v \left(\frac{g_{j+1}^n - g_{j-1}^n}{2\Delta x} \right) + \frac{1}{2} \left(\frac{g_{j+1}^n - 2g_j^n + g_{j-1}^n}{\Delta t} \right)$$

$$g_j^{n+1} = g_j^n - v\Delta t \left(\frac{g_{j+1}^n - g_{j-1}^n}{2\Delta x} \right) + \frac{(\Delta x)^2}{2} \left(\frac{g_{j+1}^n - 2g_j^n + g_{j-1}^n}{(\Delta x)^2} \right) \quad (10)$$

and this is exactly the FTCS representation of the PDE

$$\frac{\partial g}{\partial t} = -v \frac{\partial g}{\partial x} + \frac{(\Delta x)^2}{2\Delta t} \nabla^2 g \quad (11)$$

$$g_j^{n+1} = g_j^n - (v\Delta t) \frac{\partial g}{\partial x} + \frac{(\Delta x)^2}{2} \frac{\partial^2 g}{\partial x^2} \quad (12)$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2}$ in one-dimension. The second term in R.H.S. of equation (12) is a diffusion term to the equation which is in the form of Navier-Stokes equation for viscous fluid flow with dissipative term. Hence, the Lax scheme is said to have numerical dissipation or numerical viscosity. In PDE based image processing, this diffusion term produces more and more smoother and noiseless version of image as the time t goes by during the evolution of image with initial condition as the noisy image.

Therefore, from equation (1, 2), the general PDE for recovering the motion blurred image in x-direction with the initial condition of this PDE as the observed blurred image $g(x, y, t)$ reads

$It = g(x, y)$: Initial Blurred Image

$$It = It - (v\Delta t) \frac{\partial g}{\partial x} + \frac{(\Delta x)^2}{2} \frac{\partial^2 g}{\partial x^2} \quad (13)$$

Equation (13) is the desired Image Restoration Model.

IV. The Horizontal Deblurring Algorithm

The Algorithm for this scheme is as follows:-

1. Read the original image of size $m \times n$.
2. Introduce the motion blur in x direction to get $g(x, y, t)$ or alternatively we can directly have the blurred image $g(x, y, t)$. $It = g(x, y)$: Initial Image
3. Set $dx=0.1$, $dt=0.1$
4. for $t=1$: iterations

$$I_t = I_t - (v\Delta t) \frac{\partial g}{\partial x} + \frac{(\Delta x)^2}{2} \frac{\partial^2 g}{\partial x^2}$$

// Evolves the solution after n iterations
end

5. Display the image

V. The Vertical Deblurring Algorithm

The Algorithm for this scheme is as follows:-

1. Read the original image s of size $m \times n$.
2. Introduce the motion blur in y direction to get $s(y, x, t)$ or alternatively we can directly have the blurred image $s(y, x, t)$.

$I_d = s(y, x)$: Initial Image

3. Set $dy=0.1, dt=0.1$

4. for $t=1$: n iterations

$I_d = I_d - (v\Delta t) +$

// Evolves the solution after n iterations
end

5. Display the image

VII. The Combined Deblurring Algorithm

The Algorithm for this scheme is as follows:-

1. Read the original image K of size $m \times n$.
2. Filter the image K to Produce blurred version $h(x, y)$ by introducing motion in x -direction.
3. Filter $K(x, y)$ to get final version $K(x, y)$ by introducing motion blur in y -direction $K(x, y)$ is the final blurred image with motion introduced in both x and y directions).

Initial Image $I = K(x, y)$

4. Set $dx=0.1, dt=0.1, no_iterations=50, =1$

For $t=1$: no_iterations

$I = I - (\Delta t) +$

5. $R = I$

Set $dy=0.1, dt=0.1, num_iterations=50, =1$

For $t=1$: num_iterations

$R = R - (\Delta t) +$

end

6. Get R and display as final deblurred Image.

VIII. Results

All the algorithms horizontal, vertical and combination of horizontal and vertical are implemented in MATLAB 2010 and tested for various set of test images for various sizes of motion blurs e.g. 10 pixels, etc in x -direction, y -direction and combination of both the directions and the results are found to be satisfactory. For restoration of images, only the blurred image is required. In horizontal algorithm the blurred image $g(x, y, t)$ is processed for several iterations until the desired result is obtained. Its result is shown in the fig. 1.

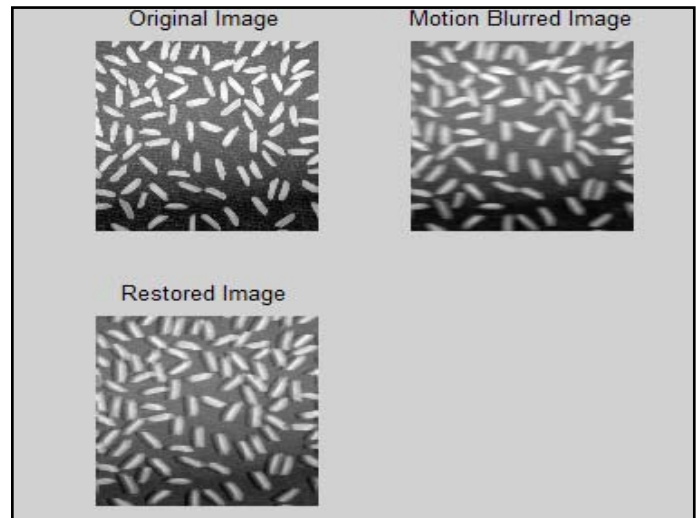


Fig. 1: Result of Horizontal Deblurring Algorithm

In vertical algorithm the blurred image $s(x, y, t)$ is processed for several iterations until the desired result is obtained.

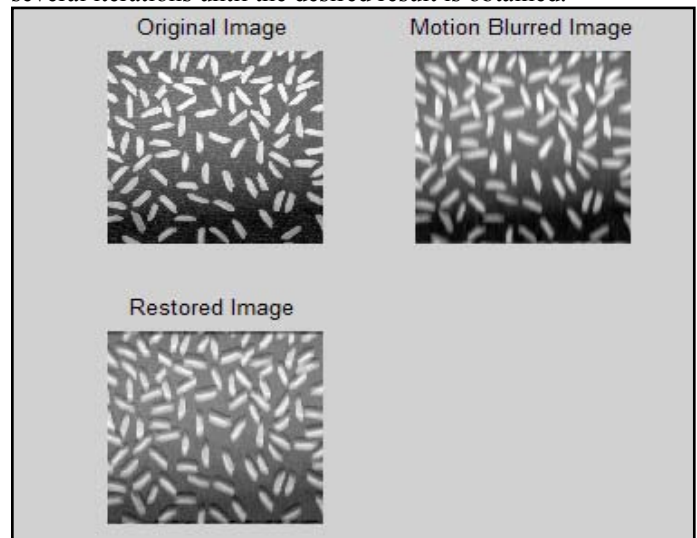


Fig. 2: Result of Vertical Deblurring Algorithm

In combined algorithm the blurred image $K(x, y, t)$ is processed for several iterations until the desired result is obtained. It has been tested by experimentation that approximately at 50 iterations the desired result is obtained i.e. de-blurred and smooth image. The Following figs. of rice shows the result of proposed scheme.

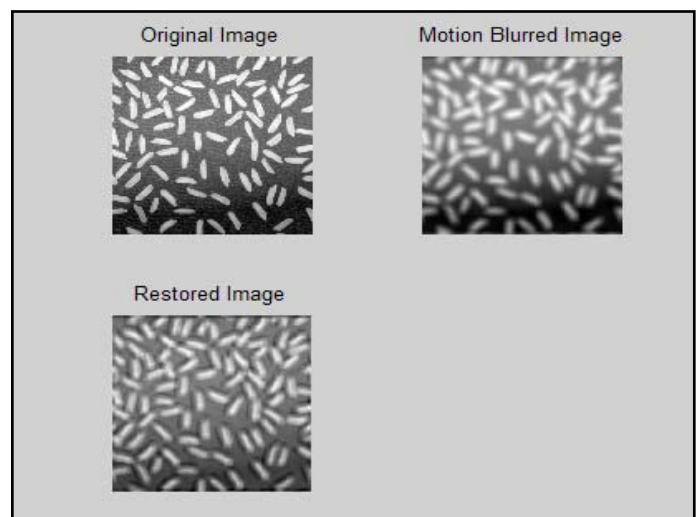


Fig. 3: Result of Combined Deblurring Algorithm

IX. Conclusion and Future Scope

In this paper horizontal ,vertical and combined image deblurring algorithm are studied. We have shown that our vertical algorithm of image deblurring gives better result than horizontal or combined deblurring algorithm.

This deblurring work can be further extended on real images, where lot of other aspects including retaining color information and effective noise suppression need additional concern along with basic deblurring.

References

- [1] Arfken, G., "Partial Differential Equations of Theoretical Physics", §8.1 in Mathematical Methods for Physicists, 3rd ed. Orlando, FL: Academic Press, pp. 437-440, 1985
- [2] Wiener, Norbert, "Extrapolation, Interpolation, and Smoothing of Stationary Time Series", New York: Wiley.
- [3] J. Biemond, A. K. Katsaggelos, "A New Iterative Restoration Scheme for Noisy Blurred Images", Proc. Conf. on Math. Methods in Signal Processing, pp. 74-76, Aachen, W. Germany, September 1984.
- [4] M. R. Banham, A. K. Katsaggelos, "Spatially Adaptive Wavelet-Based Multiscale Image Restoration", IEEE Trans. Image Processing, Vol. 5, No. 4, pp. 619-634, April 1996.
- [5] R. G White, R. A Schowengerdt, "Effect of point spread functions on precision step edge measurement", J. Opt Soc. Am. A-Optics 11, 2593-2603, 1994.
- [6] William H. Richardson, "Bayesian-based iterative method of image restoration", Journal of the Optical Society of America, Vol. 62, No. 1, 55-59, January 1972.
- [7] L.B. Lucy, "An iterative technique for the Rectification of observed images", The Astronomical Journal, Vol. 79, No. 6, pp. 745-754, June 1974.