ISSN : 2229-4333(Print) | ISSN : 0976-8491(Online)

Hybrid Threshold Technique for Speckle Noise Reduction using wavelets for Grey scale images

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Abstract

Image de-noising has become an essential task for many researchers, in Image processing. Mainly ultrasound images contain speckle noise which degrades the quality of the images. Eliminating such kind of noise is an important preprocessing task. In this paper, a hybrid method is proposed for removing speckle noise from the image. Proposed method consist of two wavelet thresholding techniques: first technique by using statistical method and second technique based on bayes threshold. Result of both method is averaged and apply threshold for soft thresholding.for post processing wiener filter is used. Experimental results on several test images were taken where; proposed method yields significantly better image quality and better Peak Signal to Noise Ratio (PSNR) on high noise. Quantitative and qualitative comparisons of the results obtained by the proposed method with the results achieved from the other speckle noise reduction techniques demonstrate its better performance for speckle reduction.

Keywords

DWT, Speckle noise, Weiner filtering, Wavelet thresholding.

I. Introduction

An image is often corrupted by noise since its acquisition or transmission. As noise is very difficult problem in the field of image processing. This problem has existed for a long time and yet there is no good solution for it. The main occurrences of noise in a digital image may arise during image acquisition and image transmission. There are basically two types of noise viz. additive noise and multiplicative noise. Depending upon the nature of noise, there are various several approaches for removal of noise from an image. The goal of de-noising is to remove the noise while retaining as much as possible the important signal features of an image. In this paper we deal with the speckle noise which is multiplicative in nature. The wavelet representation naturally facilitates the construction of such spatially adaptive algorithms. It compresses essential information in a signal into relatively few, large coefficients, which represent image details at different resolution scales. In recent years there has been a fair amount of research on wavelet thresholding and threshold selection for signal and image denoising [4, 5, 6, 7, 8, 9], because wavelet provides an appropriate basis for separating noisy signal from image signal.

A. Model for Speckle noise

The mathematical expression for a signal observed at point p whose coordinates (x,y) in the image is as follows :

$$O(x,y) = \sum\sum e(x_i, y_j)h(x - x_i, y - y \qquad (1)$$

Where e(x,y) : is signal received by the sensor, h is the impulse response of the acquisition system. [14] The intensity I(x,y) at this point can be stated in a multiplicative form as :

$$I(x, y) = | O(x, y) |^2 = e(x, y)^2 \times u(x, y) \qquad (2)$$

where u(x,u) is noise independent from the useful signal. In the following the model used for the ultrasound image is:

$$g(x, y) = f(x, y) \times u(x, y) \qquad (3)$$

where g is the observed intensity of the image and f is the free noise intensity. Within homogenous regions this model offers a good approximation. To address the multiplicative nature of speckle noise, Jain developed a homomorphic approach. An appropriate method for speckle reduction is one which enhances the signal to noise ratio while preserving the edges and lines in the image, So the Ist step is to transform the multiplicative noise model into additive noise model by taking logarithms. Also the additive model transforms into the multiplicative one by taking the exponentiation which is clear by through this example.

Equation (3) becomes by take the logarithm

$$\text{Log}(g) = \log(f \times u) = \log(f) + \log(u) \qquad (4)$$

By considering the additive model of a noise is

$$g = f + u \qquad (5)$$

$$e^g = e^{f+u} = e^f . e^u \qquad (6)$$

A trade of between the removed noise and the blurring in the image always exist. The presence of speckle noise in images tends to reduce the image resolution and contrast, thus disgrace the image quality. There are many methods which are based on different thresholding techniques for speckle noise reduction. The goal of an image de-noising algorithm is to recover the clean image from its noisy version by removing noise and retaining as much as possible the image information.

B. Previous work

In speckle filtering a kernel is being moved over each pixel in the image and applying some mathematical calculation by using these pixel values under the kernel and replaced the central pixel with calculated value. The kernel is moved along the image only one pixel at a time until the whole image covered. By applying these filters smoothing effect is achieved and speckle noise has been reduced to certain extent.

Lee filter:[18] The lee filter is basically used for speckle noise reduction. The lee filter is based on the assumption that the mean and variance of the pixel of the interest is equal to the local mean and variance of all pixels within the moving kernel. The formula for the lee filter for speckle noise reduction is given as:

$$\hat{R}(t) = I(t)W(t) + \bar{I}(t)(1 - W(t)) \qquad (7)$$

Where $W(t) = 1 - \frac{c^2{}_u}{c^2{}_1(t)}$ is the weighted function and

$$C_u = \frac{\sigma_u}{u};\ C_1(t) = \frac{\sigma_1(t)}{I(t)} \qquad (8)$$

are the various coefficients of the speckle u(t) and the image I(t), respectively.

Kaun filter:[17] In this filter given kaun et al., the multiplicative noise model is first transformed into a signal-dependent additive noise model. Then the MMSE criterion was applied to this model. The resulting filter has the same form as the lee filter but with the different weighting function which is given as:

$$W(t) = \frac{1 - \frac{C_u^2}{C_i^2(t)}}{1 + C_u^2} \qquad (9)$$

The Lee and Kuan filters have the same formation, although the signal model assumptions and the derivations are different. Essentially, both the Lee and Kuan filters form an output image by computing a linear combination of the center pixel intensity in a filter window with the average intensity of the window. So, the filter achieves a balance between straightforward averaging (in homogeneous regions) and the identity filter (where edges and point features exist). This balance depends on the coefficient of variation inside the moving window.

Frost Filter:[19] The Frost filter also strikes a balance between averaging and the all-pass filter. In this case, the balance is achieved by forming an exponentially shaped filter kernel that can vary from a basic average filter to an identity filter on a point wise, adaptive basis. Again, the response of the filter varies locally with the coefficient of variation. In case of low coefficient of variation, the filter is more average-like, and in cases of high coefficient of variation, the filter attempts to preserve sharp features by not averaging.

Srad Filter:[5] Another method has been developed with a nonlinear anisotropic diffusion technique, speckle reducing anisotropic diffusion, for removing multiplicative noise in imagery. Speckle reducing anisotropic diffusion has been emerged as a tool for reducing speckle with regional feature neatly enhanced. The method relies on the instantaneous coefficient of variation (ICOV) edge detector as a controller of diffusion rate near edges of regional structures.

Wavelet Filter:[10] wavelet transform because of its multi-resolution and sparse representations, has been shown to be a powerful tool to achieve these goals. The interest in wavelet based de-noising evolved from the seminal work on signal de-noising via wavelet thresholding or shrinkage of Donoho and Johnstone in the additive white Gaussian noise setting [10], [12]. These authors have proposed two thresholding strategies in which each wavelet coefficient is compared against a threshold; if the coefficient is smaller than the threshold, it is set to zero, otherwise it is kept (hard thresholding) or modified (soft thresholding) [15].

Bayes method: [22] Bayes Shrink was proposed by Chang, Yu and Vetterli. The goal of this method is to minimize the Bayesian risk, and hence its name, Bayes Shrink.The Bayes threshold, is defined as

$$t_b = \frac{\sigma^2}{\sigma_s} \tag{10}$$

Where σ^2 is the noise variance and σ_s is the signal variance without noise. The noise variance σ^2 is estimated from the sub band HH by the median estimator shown in equation (14). From the definition of additive noise we have **W(x,y) = s(x,y) + n(x,y).** Since the noise and the signal are independent of each other, it can be stated that

$$\sigma_W^2 = \sigma_S^2 + \sigma^2 \tag{11}$$

σ_W^2 can be computed as shown below:

$$\sigma_W^2 = \frac{1}{n^2}\sum_{x,y-1}^{n} W^2(x,y) \tag{12}$$

The variance of the signal, σ2 s is computed as

$$\sigma_s = \sqrt{max(\sigma_W^2 - \sigma^2, 0)} \tag{13}$$

$$\sigma = \frac{Median\,|Y_{i,j}|}{0.6745}, \tag{14}$$

$$Y_{i,j} \in subband\ \ HH1$$

With σ^2 and σ_W^2, the Bayes threshold is computed from Equation (12). Using this threshold, the wavelet coefficients are threshold at each band.

The use of wavelet transform for signal de-noising has been started in last decade. Wavelets capability to give detail spatial-frequency information is the main reason for this investigation. This property promises a possibility for better discrimination between the noise and the real data. Typically, noise is characterized by high spatial frequencies in an image, Fourier based methods usually try to suppress high frequency components, which also tends to affect sharpness of edges. Since the wavelet transform provides good localization in both spatial and spectral domains, low pass filtering is inherent to this transform.

We describe here an efficient hybrid thresholding technique for de-noising the speckle, by decomposing the corrupted images into the respective sub-bands by applying the discrete wavelet transform on the input noisy image with combination of Bayes thresholding. On the other hand, because of the different nature of the speckle noises, different methods are used to suppress each. For speckle noise reduction, some other filters such as Lee [18] and Kuan et al. [17], Frost [19], Srad [5] produces better result but at the cost of blur image. These filters are based on local noise statistics in the spatial domain and can effectively reduce speckle in homogeneous areas. However, these filters do not perform well in the edges. Recently some attempts have been made to reduce the speckle noise using wavelet transform. The existing methods don't determine the thresholding value and the thresholding function simultaneously. Many of the existing methods need to consider some priori assumptions about the statistical distribution of subbands of noise free wavelet coefficients. In this paper, hybrid technique is used by applying computing two threshold values by different method one produced by a new non linear thresholding function proposed for image de-noising in the wavelet domain and other by bayes thresholding. This function has some advantages over classical methods and produces better results in speckle noise reduction.

II. Introduction to WAVELETS

A. Wavelet

Wavelet means a "small wave". The smallness refers to the condition that the window function is of finite length (compactly supported). A wave is an oscillating function of time or space and is periodic. In contrast, wavelets are localized waves. The difference between wave and wavelet are shown in Fig. below 2.1. They have their energy concentrated in time and are suited to analysis of transient signals. In wavelet analysis the signal to be analyzed is multiplied with a wavelet function and then the transform is computed for each segment generated. The Wavelet Transform, at high frequencies, gives good time resolution and poor frequency resolution, while at low frequencies. An arbitrary signal can be analyzed in terms of scaling and translation of a single mother wavelet function (basis). Wavelets allow both time and frequency analysis of signals simultaneously because of the

ISSN : 2229-4333(Print) | ISSN : 0976-8491(Online)

fact that the energy of wavelets is concentrated in time and still possesses the wave-like (periodic) characteristics [18, 19].

Properties of wavelets: 'Regularity' is defined as if r is an integer and a function is r-time continuously differentiable at x=0, then the regularity is r. If r is not an integer, let n be the integer such that n<r<n+1, then function has a regularity of r in x0 if its derivative of order n resembles $(x-x0)^{r-n}$ locally around x0. This property is useful for getting nice features, such as smoothness, of the reconstructed signals.

- The support of a function is the smallest space-set (or time-set) outside of which function is identically zero.
- The number of vanishing moments of wavelets determines the order of the polynomial that can be approximated and is useful for compression purposes.

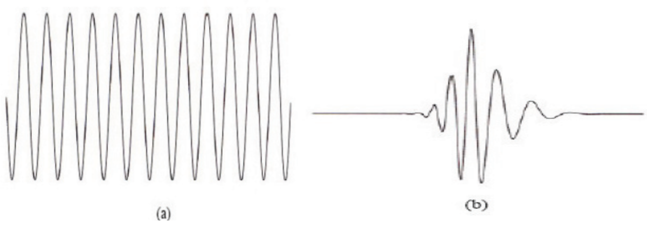


Fig. 2.1: Difference between Wave and Wavelet (a) wave (b) wavelet.

B. Wavelet Domin Noise Filtering

Recently there has been significant investigations in medical imaging area using the wavelet transform as a tool for improving medical images from noisy data. Wavelet denoising attempts to remove the noise present in the signal while preserving the signal characteristics, regardless of its frequency content. As the discrete wavelet transform (DWT) corresponds to basis decomposition, it provides a non redundant and unique representation of the signal. Several properties of the wavelet transform, which make this representation attractive for denoising, are:[20, 21]

- Multiresolution - image details of different sizes are analyzed at the appropriate resolution scales
- Sparsity - the majority of the wavelet coefficients are small in magnitude.
- Edge detection - large wavelet coefficients coincide with image edges.
- Edge clustering - the edge coefficients within each sub band tend to form spatially connected clusters

During a two level of decomposition of an image using a scalar wavelet, the two-dimensional data is replaced with four blocks. These blocks correspond to the sub bands that represent either low pass filtering or high pass filtering in each direction. The procedure for wavelet decomposition consists of consecutive operations on rows and columns of the two-dimensional data. The wavelet transform first performs one step of the transform on all rows. This process yields a matrix where the left side contains down sampled low pass coefficients of each row, and the right side contains the high pass coefficients. Next, one step of decomposition is applied to all columns; this results in four types of coefficients, HH, HL, LH and LL.

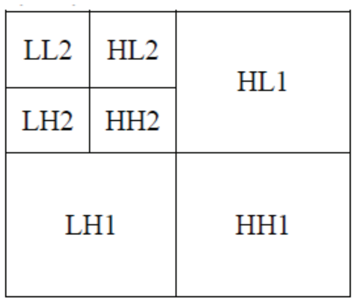


Fig. 2.2: Two-Level Image decomposition by using DWT

The LL sub-band is the result of low-pass filtering both the rows and columns and it contains a rough description of the image as such. Hence, the LL sub-band is also called the approximation sub-band. The HH sub-band is high-pass filtered in both directions and contains the high-frequency components along the diagonals as well. The HL and LH images are the result of low-pass filtering in one direction and high-pass filtering in another direction. LH contains mostly the vertical detail information that corresponds to horizontal edges. HL represents the horizontal detail information from the vertical edges. All three subbands HL, LH and HH are called the detail subbands, because they add the high-frequency detail to the approximation image [19].

III. Proposed Algorithm

A. Algorithm

This part of algorithm contains the stepwise, detailed methodology that is followed while de-noising images using wavelet transforms. The proposed algorithm is described as follows.

1 Read the original standard image in tiff format for example lena.tif, wom.tif. The size of the standard image is taken as 256×256.

2 Speckle noise is added to standard images with different standard deviations. Speckle noise is multiplicative noise, so it is first converted into additive noise.

3 Transform the multiplicative noise model into an additive one by taking the logarithm of the original speckled data.

4 Log I(x, y) = log S(x, y) + log$\eta(x, y)$.

5 Perform the DWT of the noisy image up to 2 levels (L=2) to obtain seven sub bands, which are named as LL1, HH1, LH1, HL1, HH2, LH2, HL2, LL2.

6 Compute the threshold values for each sub band, except the LL2 band using Step7 and Step 10 equations.

7 TH1 = 2.5*C, $\sigma_1 - (|AM - GM|)$ after finding out the following terms.

8 Obtain the noise variance using the equation $\sigma_1 = \frac{Median|Y_{i,j}|}{0.6745}$, $Y_{i,j}$ ε sub band each coefficient.

9 Find the term C for each sub band using relation given in equation C = $2^{(L-K)}$ and calculate term $|AM - GM|$ using the equation $\frac{1}{M^2}\sum_{i=1}^{M}\sum_{j=1}^{M}X(i,j)and\left[\prod_{i=1}^{M}\prod_{j=1}^{M}X(i,j)\right]^{\frac{1}{M^2}}$

10 $Th2 = \begin{cases} \sigma_x = 0 & \max(abs(X) \\ else & \sigma^2/\sigma_x \end{cases}$

after finding out following terms.

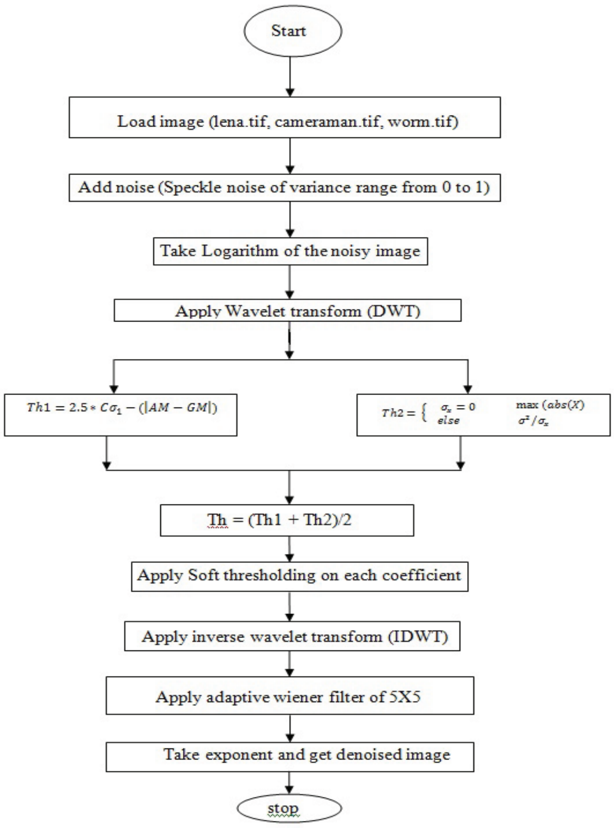
11 Obtain σ_x and σ^2 by following formula's:

$$\sigma_x = \sqrt{\max\left(\frac{\sum X(i,j)^2}{length(X)} - \sigma^2, 0\right)}$$

$$\sigma = \frac{Median\ |Y_{i,j}|}{0.6745}, Y_{i,j}\ \epsilon\ subband\ \ HH1$$

12 Compute the final Th as average of Th1 and Th2.
Th = (Th1 + Th2) / 2 ;
13 Threshold the all sub band coefficients (except LL2) using soft thresholding technique given in equation 16, by substituting the threshold value obtained in Step12. Perform the inverse DWT to reconstruct the denoised image by using the soft thresholding technique.
14 Applying wiener filter [4] to above denoised image taking window size of 5×5.
15 Take exponent and get denoised image.

B. Flow chart of algorithm



Step 13 threshold selection
For a thresholding algorithm to be really effective, it should preserve the edges information and other relevant information about an image. There are basically two types of general thresholding algorithm.
Global thresholding algorithms
Local or adaptive thresholding algorithms
In global thresholding, a single threshold for the entire image pixel is used. When the pixel values of the components and that of background are fairly consistent in their respective values over the entire image, global thresholding could be used.
In adaptive thresholding, different threshold values for different local areas are used. Thresholding is further of two types:
- Hard thresholding
- Soft thresholding

Hard Thresholding: Basically, it threshold the detail coefficients. That is, select and apply a threshold to the detail coefficients (horizontal, vertical, diagonal) of the image after applying the discrete wavelet transform. It refers to the procedure where the input elements with absolute value lower than the set threshold value, are set to zero.

I(x, y) = $\sigma_1 - ($
(15)

Soft Thresholding: it involves first setting to zero the elements whose absolute values [15] are lower than the threshold and then scaling the non-zero coefficients towards zero. Soft thresholding eliminates the discontinuity that is inherent in hard thresholding.

$$I(x,y) = \begin{cases} (x,y) - T, f\ (x,y) \geq T \\ (x,y) + T, f\ (x,y) \leq -T \\ 0, f\ |(x,y)| \leq T \end{cases} \tag{16}$$

Hard thresholding is "keep or kill" procedure and is more intuitively appealing and also it introduces artifacts in the recovered images. But soft thresholding is more efficient and it is used for the entire algorithm because after applying soft thresholding technique we yield visually more pleasing images [18, 19]. Also hard threshold exhibits the unwanted accessional fluctuation and soft threshold attenuates the range of the wavelet coefficients and smoothes the signal or images as shown in below fig. 3.1.

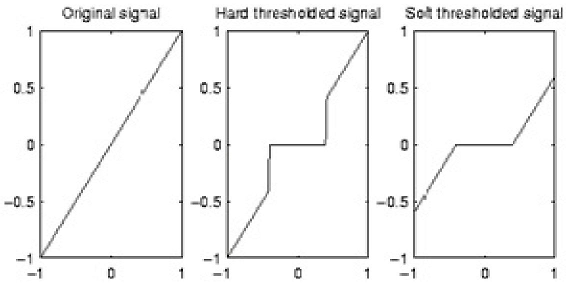


Fig.3.1 (a): Hard thresholding of signal (b) Soft thresholding of signal

IV. Experimental Results And Discussion

The above algorithm has been applied on several images like Lena, Barbara, Cameraman of size 256×256, at different Speckle noise level of different standard deviation σ = 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.09. In this algorithm, we used 'Daubechies' wavelet (Db4), because of the least asymmetric compactly supported wavelet at two level of decomposition [20].The results in this algorithm are analyzed both qualitatively and quantitatively. For quantitative analysis two parameters are used MSE (Mean Square Error), PSNR (peak signal to noise ratio) are calculated for all the standard images with their noisy and denoised counterparts, respectively.
PSNR – PSNR stands for the peak signal to noise ratio. It is an engineering term used to calculate the ratio between the maximum possible power [21] of a signal and the power of corrupting noise that affects the fidelity of its representation. Because many signals have a very wide dynamic range, PSNR is usually expressed in terms of the logarithmic decibel scale. It is calculated as the following:

$$PSNR = 10\log_{10}\frac{255\times255}{MSE} \tag{17}$$

$$\text{MSE} = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\left[X(i,j) - Y(i,j)\right]^2 \qquad (18)$$

Where X and Y are the original and noisy or denoised image respectively. M and N represent the width and height of image.

Table 1: PSNR output for variance =0.02

Algorithm	Camera man	Lena	Worm
Srad Method	26.65	26.52	29.57
Frost Method	26.12	25.49	26.94
Kaun Method	25.47	24.86	25.95
Lee Method	25.47	24.86	25.95
GGD Method	24.29	24.66	23.47
Bayes Method	26.18	27.36	30.79
Proposed Method	**27.23**	**28.24**	**32.53**

Table 2: MSE output for variance =0.02

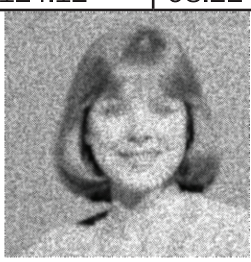
Algorithm	Cameraman	Lena	Worm
Srad Method	141.68	145.96	72.35
Frost Method	160.22	185.09	132.49
Kaun Method	186.11	214.06	166.58
Lee Method	186.11	214.06	166.58
GGD Method	243.98	224.05	294.87
Bayes Method	158.00	120.37	54.68
Proposed method	124.12	98.22	36.59

Table 3: PSNR output for variance =0.09

Algorithm	Cameraman	Lena	Worm
Srad Method	23.05	23.19	23.37
Frost Method	21.37	21.29	20.79
Kaun Method	23.11	22.68	22.85
Lee Method	23.11	22.68	22.85
GGD Method	18.38	18.59	17.22
Bayes Method	20.46	22.70	25.69
Proposed method	24.48	24.56	25.74

Table 4: MSE output for variance =0.09

Algorithm	Cameraman	Lena	Worm
Srad Method	141.68	145.96	72.35
Frost Method	160.22	185.09	132.49
Kaun Method	186.11	214.06	166.58
Lee Method	186.11	214.06	166.58
GGD Method	243.98	224.05	294.87
Bayes Method	158.00	120.37	54.68
Proposed method	124.12	98.22	36.59



ISSN : 2229-4333(Print) | ISSN : 0976-8491(Online)

Srad Method Frost Method

GGD Method Kaun Method

Bayes Method Proposed Method

Fig. 4.1: Visual results for speckle reduction for O' = 0.09 on worm.tif

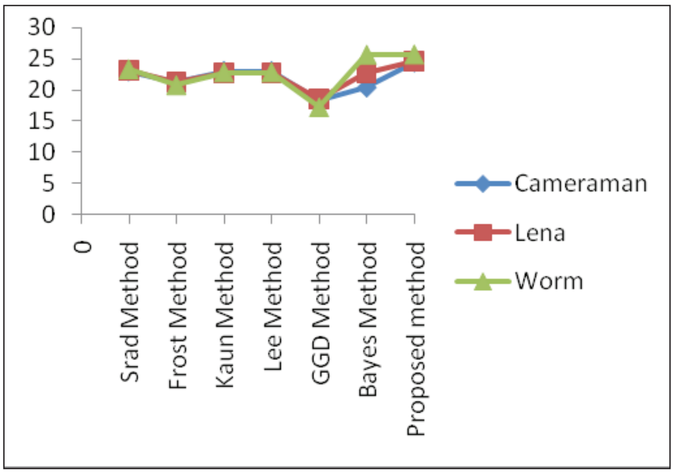


Fig.4.2 : Graphical PSNR results for speckle reduction for worm.tif

V. Conclusion

In this work we studied some of the traditional techniques for speckle noise reduction like lee, frost, SRAD, kaun filters, wiener, median filtering and also wavelet based techniques studied and implemented in matlab version R2010a. The main goal of the speckle noise reduction is to satisfy the important factors during image enhancement: edge preservation, speckle noise removal, better smoothing of an image. In this paper we try to explore the hybrid technique based on wavelet based method for speckle noise reduction with combination of bayes thresholding method and performance of this method is compared with existing traditional techniques in terms of PSNR (peak signal to noise ratio) and visual results. It has been observed that combination of this method does perform better than the existing techniques. In wavelet based techniques edge preservation is also good and better speckle noise suppression.

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