Sevcik's Fractal Based Dimensionality Reduction of Hyper-Spectral Remote Sensing Data

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Abstract
Recent advancements in Remote sensing has led the way for the development of Hyper spectral sensors. Hyper spectral remote sensing is a new technology that is currently being used by researchers and scientists with regard to the detection and identification of mineral, vegetation, man-made materials and other features. These sensors are far more superior as they collect information in a very narrow contiguous wavelength interval. Hyper spectral data carries information of the features in ten to thousand bands. This wealth of data is hard to exploit, as it needs very high computational complexity to store and to process this data. Hyper spectral data are also highly correlated, so special attention is needed to reduce the dimensions of hyper spectral data.
The spectral reflectance of any pixel depends on the characteristics of a land cover class. The fractal dimension of the spectral reflectance curve (SRC) of any pixel can thus be calculated. Based on this, fractal based method can thus be employed to reduce the dimensionality of hyper spectral remote sensing data. The fractal dimension of SRC is calculated by sevcik's method. As a case study, AVIRIS 224 band data is used and processed in MATLAB 7.8.0. The study shows that the dimensionality of hyper spectral remote sensing data can be reduced with minimal data loss.

Keywords
Hyper spectral data, SRC, Fractal dimension, AVIRIS, MATLAB

I. Introduction

A Hyper spectral image consists of about a hundred or more contiguous spectral bands. Hyper spectral remote sensing, also known as imaging spectroscopy, is a relatively new technology that is currently being investigated by researchers and scientists with regard to the detection and identification of minerals, terrestrial vegetation, and man-made materials and backgrounds. Hyper spectral data sets are generally composed of about 100 to 200 spectral bands of relatively narrow bandwidths (5-10 nm), whereas, multispectral data sets are usually composed of about 5 to 10 bands of relatively large bandwidths (70-400 nm). These sensors are far more superior as they collect information in a very narrow contiguous wavelength interval.

Hyper spectral imagery provides an opportunity for more detailed image analysis. For example, using hyper spectral data, spectrally similar materials can be distinguished, and sub-pixel scale information can be extracted.

Hyper spectral data utilizes a large portion of the electromagnetic spectrum and captures a huge amount of information within the region. For example, a 220 band Hyperion data at 30 m pixel size covering an area of 30 km by 30 km, digitized at 11 bits (stored at 16 bits) has a data volume of 220x1000x1000x2 bytes.

A. Processing of Hyper spectral Data

Hyper spectral data is capable of collecting imagery in hundreds of spectrum bands. However, the increase in the number of bands is both a blessing and a curse. The large number of bands provides the opportunity for more materials to be discriminated by their respective spectral response. However, this large number of bands is the characteristic which leads to complexity in analysis techniques. Hyper spectral sensors utilize a large portion of electromagnetic spectrum and capture a huge amount of information in this region. The most challenging problem of working with hyper spectral data is that it contains a huge amount of information and therefore working with this data involves a tremendous computational burden.

Hyper spectral data are highly correlated. It is generally observed that adjacent band shows a high correlation value than bands of different wavelength region. This proves that the information contained by a hyper spectral data is redundant.

Another problem associated with hyper spectral data is 'curse of dimensionality' or the 'Hughes phenomenon' which is very likely for hyper spectral data having a large number of dimensions.

So, working with original dimensionality involves many problems and the huge volume of hyper spectral data is always associated with heavy computational burden, therefore there is a need for reducing its dimensionality. Since hyper spectral data is mostly redundant, i.e. highly correlated, hence it can be represented in lower dimensional space without losing much of its significant information.

B. Dimensionality Reduction

Since working with original dimensionality involves many problems and the huge volume of hyper spectral data is always associated with heavy computational burden, therefore there is a need for reducing its dimensionality. Since hyper spectral data is mostly redundant, i.e. highly correlated, hence it can be represented in lower dimensional space without losing much of its significant information.

III. Theory of Fractal Dimension

According to Euclidean Geometry, the dimension of any object on the ground may be either 1 , 2 or 3, i.e. the dimension of an object is an integer quantity. Thus dimension 1 is for a linear feature, 2 for area and 3 for volumetric feature. However, fractal dimension is a new concept coined by Mendelbrot [ ], according to which an irregular feature cannot be 1, 2 or 3 rather it is fractional. Fractal mathematics states that a straight line will have a dimension 1 while a curved feature will have a dimension between 1 and 2. Fractal dimension of a feature always exceeds its Euclidean dimension. Thus a curved line will have a dimension > 1 and < 2, a surface will have a dimension >2 and < 3 and so on.

In Euclidean Geometry, the topological dimensions are always an integer value. Thus a straight line no matter how irregular it may be will always have a dimension 1 in topological representation. On the other hand, in fractal geometry more a line becomes irregular, the closer it approaches to 2.

A. Sevcik's Fractal Dimension

Sevcik proposed the methodology of making use of Hausdorff metric for obtaining the fractal dimension of SRC (waveform) (Sevcik, 1998).

Fractal dimension of SRC at each pixel location in HS images will provide the characteristics of the object present at that location. It further results in reduction in dimensionality of N Hyper spectral bands.

The Hausdorff dimension is given by:

$$Dh = \lim \varepsilon \rightarrow 0 \, [-\ln(N(\varepsilon)] \, / \, [\ln(\varepsilon)] \quad (1)$$

Where, N (ε) is the number of open balls of radius ε needed to cover the set. In the case of a line of length L consisting of segments of length 2× ε each, then

$$N(\varepsilon) = [L/(2\varepsilon)]$$

Thus equation may be rewritten as:

$$Dh = \lim \varepsilon \rightarrow 0 \left[\frac{-\ln(L) + \ln(2\varepsilon)}{\ln(\varepsilon)}\right] = \lim \varepsilon \rightarrow 0 \left[1 - \frac{\ln(L)}{\ln(\varepsilon)}\right] \quad (2)$$

The length 'L' here represents the length of the spectral response curve.

$$L = \sum_{i=1}^{n-1} (d(i, i+1)) \quad (3)$$

Where,

$$d(i, i+1) = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_I)^2} \quad (4)$$

Sevcik proposed that the curve should be normalised within values of 0 and 1.

Transformation that normalises abscissa is given by

$$X^* = x_i / x_{\max} \quad (5)$$

where x_i is the original value of the band number and x_{max} is the maximum value at the abscissa.

Transformation that normalises the ordinate is given by

$$Y^* = \frac{y_i - y_{\max}}{y_{\max} - y_{\min}} \quad (6)$$

where y_i is the original DN value, y_{max} is the maximum DN value and ymin is the minimum DN value.

Thus the SRC is mapped into other 'n' points that belong to a unit square. This unit square may be visualised as covered by a grid of n x n cells. Taking ε=1/2N' equation (2) becomes,

$$Dh = 1 + \ln(L) \, / \, \ln(2N') \quad (7)$$

Where N' = N-1

III. Methodology

Hyper spectral data (or spectra) can be thought of as points in an N-dimensional. Let a hyper spectral data of N bands and M pixels in each band.

$$Y_{MXN} = \begin{bmatrix} y1,1 & \cdots & y1,N \\ \vdots & \ddots & \vdots \\ yM,1 & \cdots & yM,N \end{bmatrix}$$

Therefore any pixel can be expressed as a vector:

$y_k = y_{k,1}, y_{k,2}, y_{k,3}, \ldots y_{k,n}$

The SRC of each pixel can be obtained:

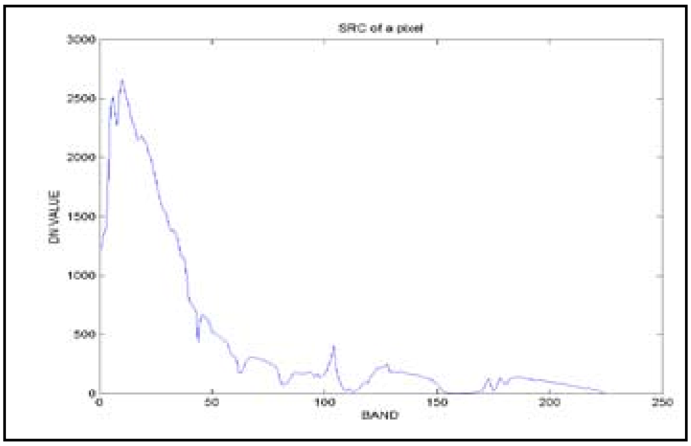

Fig. 1: Original SRC of a Pixel

At the first step all the noisy bands, bands containing no data and water absorption bands are removed and rest of the bands are used for further processing.

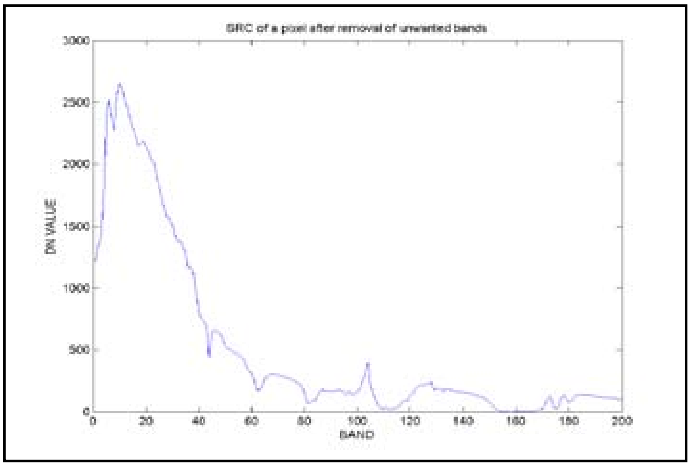

Fig. 2: SRC of a Pixel After Removal of Unwanted Bands

Even after removal of unwanted bands there are some sharp transitions in the SRC, these are also treated as noise. This will affect the nature of SRC and hence the fractal dimension values very badly. Therefore, filtering is done on the SRC so as to smoothen it out. Mean filtering is been applied on the DN values of each pixel vector so as to smoothen out the SRC of a pixel.

Method used for mean filtering is:

$$X_m = \frac{Xn+1 + Xn-1}{4} + \frac{Xn}{2} \quad (8)$$

Where, X_m is the mean DN Value obtained after applying mean filtering X_{n+1}, X_{n-1},X_n are the successive, preceding and original DN Value of the pixel.

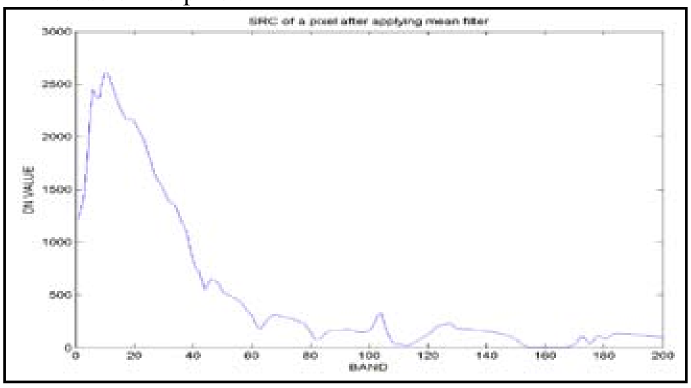

Fig. 3: SRC of a Pixel After Applying Mean Filter

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A. Pre-processing of Hyper Spectral Data

This step comprises of "smoothing" and "interpolation" of the hyper spectral data. Smoothing is an important step to get a uniform SRC. Since hyper spectral data is prone to noise, often there are sharp transitions due to the noise present in the data. As a result there would be sharp transitions in the data. Therefore, the SRC is to be smoothened.

The cubic spline fitting technique is a powerful numerical method and has been widely used in engineering and scientific computing. For example, Numerical Recipes (Press et al., 1989) provides standard subroutines, using cubic spline fitting method, for interpolating data between points.

The cubic spline interpolation technique is used to increase the number of data points available in SRC.

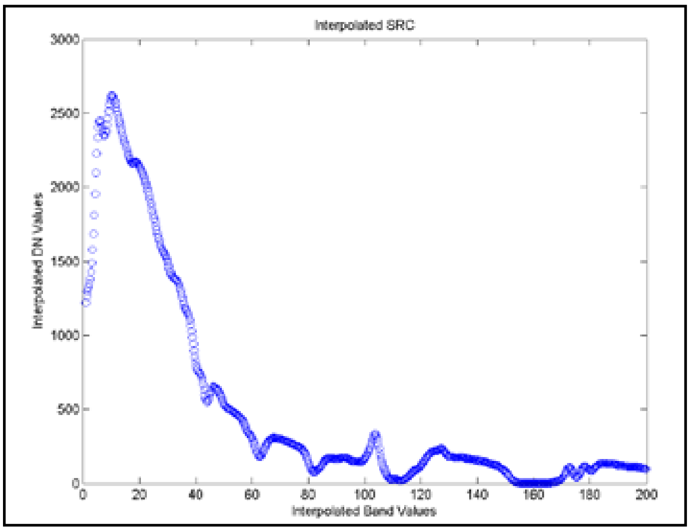

Fig. 4: Interpolated SRC

B. Computation of Fractal Dimension

This is the most important step in which the fractal dimension of the SRC is being calculated. For calculating fractal dimension from sevcik's method the first step is to normalize the abscissa and ordinate. This step is included because the curve should be normalized between 0 and 1.

Transformation that normalizes abscissa is given by:

$$X^* = {x_i}/{x_{\max}} \quad (9)$$

Transformation that normalizes the ordinate is given by:

$$y^* = \frac{y_i - y_{max}}{y_{max} - y_{min}} \quad (10)$$

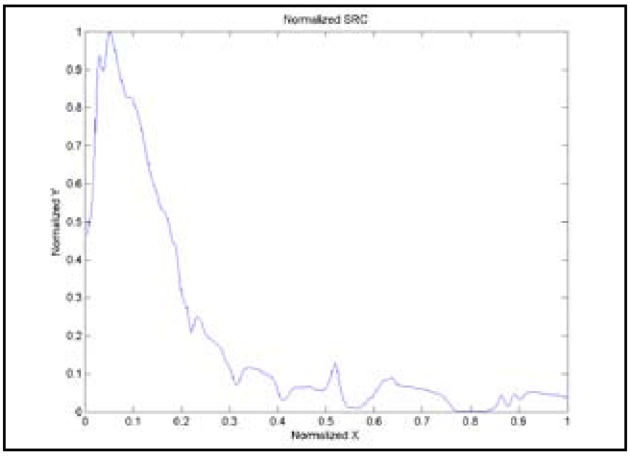

Fig. 5: Normalized SRC

In the next step the whole SRC is divided into a window size. This window is a moving window and it moves along the interpolated SRC. Each window contain certain data points in both X and Y axis.

Define a moving window of length 'a' in which the fractal dimension computation is performed. The window moves along the interpolated SRC and contain a=floor (L/n) data data-points, where 'a' is an integer value of L/n and n is the number of features required to be generated.

Once the window is defined the distance between the data points in a window is calculated.

$$L = \sum_{i=1}^{n-1} d(i, i+1) \quad (11)$$

Where,

$$d(i, i+1) = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \quad (12)$$

Now, applying the formula for calculating the dimension:

$$D_h = \lim_{N \to \infty} 1 + \frac{\ln (L)}{\ln (2 * N')}$$

N'=N-1

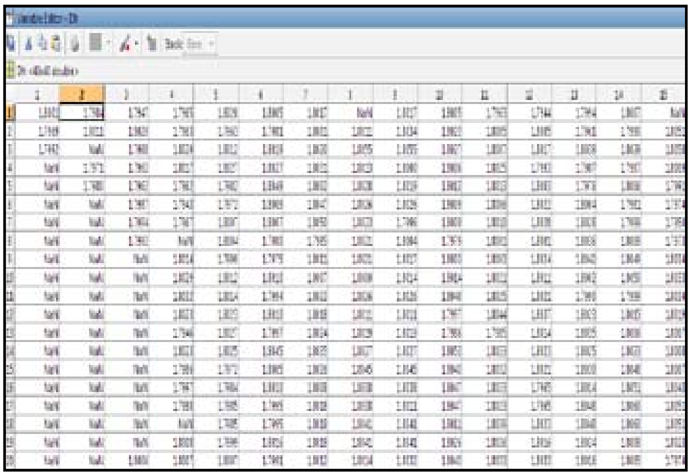

Fig. 6: Fractal Dimension Values

The calculated dimensions must lie between $1 < D_h < 2$.

C. Feature Generation

To increase the class separation between the classes, it is also important that the fractal dimensional values be multiplied with the corresponding spectral energy of each segment.

The class separation can be improved if the fractal dimension value be multiplied by the energy associated with the window by the corresponding portion of the SRC.

The energy associated with the signal in each window (E_m) is given by:

$$E_m = \sum_{j=1}^{a} Z m, j$$

Where m=1,2,3….n

$Z_{m,j}$ is the j^{th} data point value of in m^{th} window of the spline interpolated SRC.

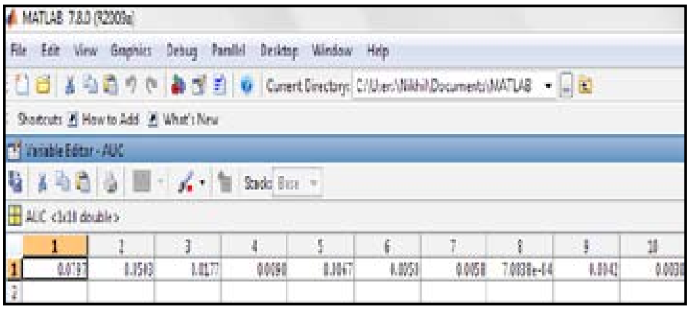

Fig. 7: Fractal Energy Values

Finally the fractal dimension values should be multiplied with the corresponding spectral energy.

$F_m = FD_m \times E_m$ for m=1,2,3…n

Thus from original N band hyper spectral data, proposed methodology generates a reduced set having n dimensions.

D. Selection of Optimum Number of Bands

The choice of optimum number of reduced bands is then carried out by making use of training and testing samples. The result of classification giving highest result is then chosen to be the reduced band "n".

IV. Data Resource and Study Area

Table 1: Data Resource

S.No	Date of Acquisition	Sensor	Resolution
1.	08-06-2011	AVIRIS	17

The size of the data is 505,365 KB. This is a 16 bit data having 1924 lines, 753 samples and 224 bands. The study area is located in USA and is known as Moffett Field.

The image has been collected over Mofett field, California by an Aircraft having aircraft ID ER2 806. The image is an ortho-corrected image.

Result

The reduced bands with highest classification accuracy obtained were 11 for the data set.

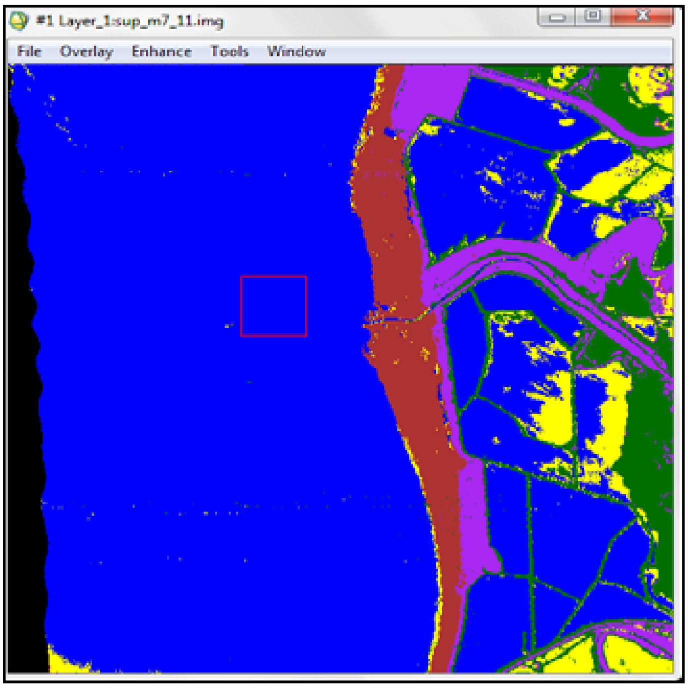

Table 2: Legends

Blue	Water
Brown	Wet Land
Violet	Dry Land
Yellow	Swamp
Green	Barren land

Table 3: Accuracy Table

Class Name	Reference Total	Classified Total	Number Correct	Producer Accuracy (%)	User Accuracy (%)
Swamp	6	8	6	100	75
Dry land	24	27	24	100	88.89
Wet land	16	19	15	93.75	78.95
Barren	31	26	21	67.75	80.77
Water	179	176	168	93.85	95.45

VI. Conclusion

The fractal dimension along different segments of the SRC bear information related to the characteristics of the SRC in terms of degree of irregularity i.e they take care of the nature of SRC very well.

To distinguish different classes from each other, information related to class separation can be taken into account by multiplying it with corresponding spectral energy value. Finally, optimum reduced dimension can be obtained by varying the number of features and finding the features which produces maximum accuracy of classification.

SRC of a pixel may be affected by noise. Thus SRC is to be smoothed using low pass filtering which reduces the noise by converting sharp transitions into smooth one.

Cubic spline interpolation is one of the most important step to increase the data sets. Sevcik's method will be affected if the data sets are less.

The result of the classification accuracy shows that SRC of various classes is one of the main characteristics to distinguish them from each other along with the nature of SRC.

The overall computational complexity of the MATLAB code is O (n^3).

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