Solving Maximum Flow and Minimum Cut Network Problems by Labeling Method

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Abstract
There are several methods available for the solution of maximum flow network problems. Labeling method is an alternative method for maximum flow network problems. The basic thing in the labeling procedure is to systematically attach labels to the nodes of a network until optimum solution is obtained. Labeling techniques can be used to solve different types of network problems. Such as shortest-path problems, maximal-flow problems, general minimal-cost flow problems etc.

Keywords
Network, Labeling Technique, Maximum Flow, Minimum Cut etc.

I. Introduction

According to Ford & Fulkerson, Max-flow Min-cut theorem for any network the value of maximal-flow from source to sink is equal to the capacity of minimal-cut. The cut with the smallest capacity is called minimal cut. Using this theorem maximal-flow in a network can be found by finding the capacity of all the cuts, and choosing the minimum capacity. Though this gives maximal value of f, it does not indicate how this flow is sent through various arcs. It is because of this thing that a different procedure known as maximal flow algorithm has been developed which is also based on max-flow min-cut theorem. The basic principle of this procedure is to find an augmenting path through which a positive flow can be sent from the source node to the sink node.

II. Labeling Algorithm

This is used to find a flow augmenting path from source to the sink. From any node i, the node j can be labeled if one of the following conditions is satisfied:

1. That is the flow in the arc(i,j) is less than its capacity
2. The flow in the arc(j,i) is greater than zero.

This process is continued till sink node is labeled and a flow augmenting path is obtained.

III. Max-flow Algorithm

1. First source node s and the sink node t have been identified
2. An initial feasible flow solution has been assumed to be equal to 0.
3. Then a flow augmenting path defined by the sequence of arcs has been determined.
4. Then maximum flow increase in the path has been determined.

Consider a Network with capacity

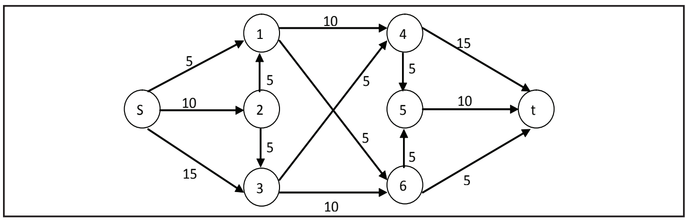

Fig. 1:

In this network first we consider a flow on each arc is zero. Then we select an augmenting path.

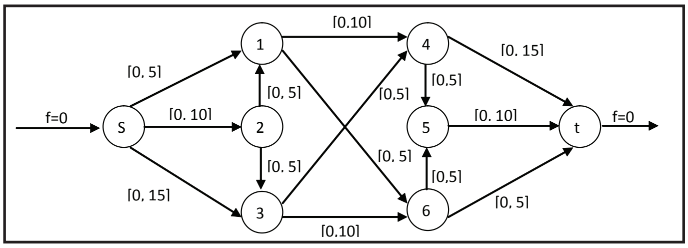

Fig. 2:

Here we select an augmenting path P1 = {s,2,3,4,5,t}. The maximum unit of flow can pass through this path is 5 units.

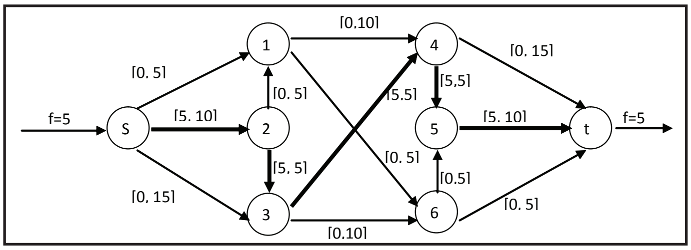

Fig. 3:

Now we again selecting the new augmenting path P2 = {s,2,1,6,5,t}. The maximum unit of flow that can be sent through this path is 5 units. Then the new configuration of network is

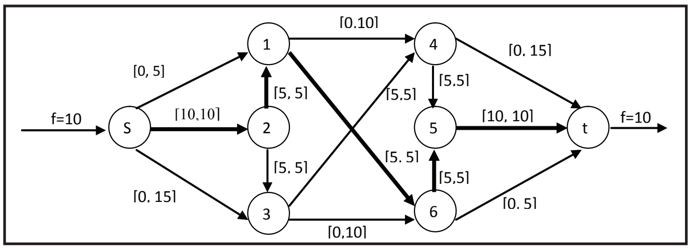

Fig. 4:

Now we again selecting the new augmenting path P3 = {s,1,4,t}. The maximum unit of flow that can be sent through this path is 5 units. Then the new configuration of network is

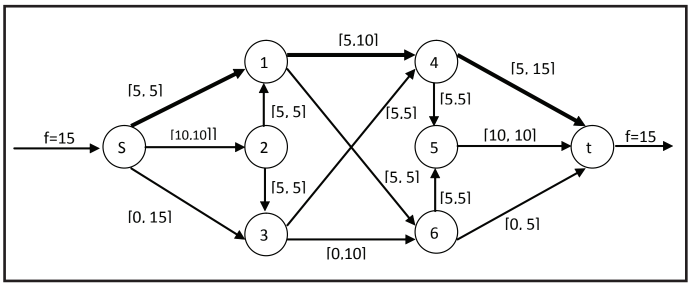

Fig. 5:

Then Selecting a new augmenting path P4 ={s,3,6,t}, has been obtained to be of 5 units.

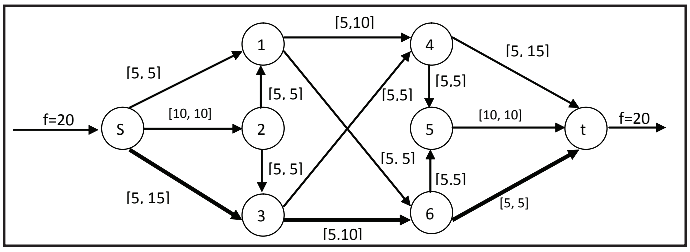

Fig. 6:

Considering the above network, the node 3 has more capacity to flow so, we can increase the flow of 5 units.

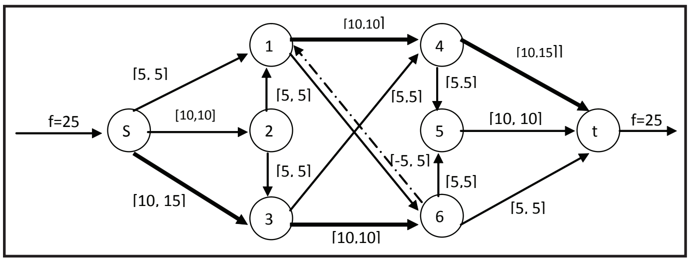

Fig. 7:

Thus the maximum flow of this network is 25. Now we consider the minimum cut

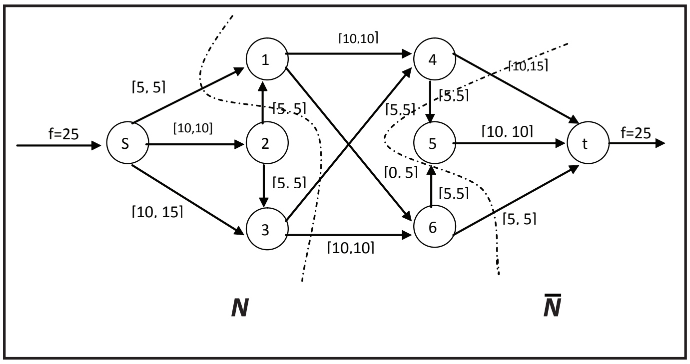

Fig. 8

Where $\bar{N}$ stands for network on left side and N stands for network on right side.

Clearly the max-flow value = min-cut value = 25

IV. Conclusion

At the present time networks, such as network of telephone lines, network of highways, railways, network of water pipelines have become indispensable. Thus the mathematical analysis of such networks has become essential.

References

[1] Ravindran, Phillips, Solberg; Operations Research (Principles and Practice); Wiley India.

[2] Narsingh Dev,"Graph theory with application to engineering science", Prentice-Hall of India Private Limited New Delhi.

[3] Ahuja, R.K, T.L.Magnanti, J.B. Orlin,"Network flows: Theory, Algorithms and Applications", Prentice-Hall ; Englewood Cliffs, NJ 1993.

[4] Ba-zaraa, M.S,J.J. Jarvis, H.D Sherali,"Linear Programming and Network flows Wiley", Newyork 1990.

[5] Murly, K.G,"Network Programming", Prentice Hall Englewood Cliffs, NJ, 1992.

[6] L.R.Ford, D.R. Fulkerson,"Maximal Flow through a Network", can.J.Math-8. The RAND Corp. Nov. 1954.