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Application of Cauchy's Residue Theorem to Solve Complex Integral using MATLAB

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Abstract

Cauchy's Residue Theorem is a powerful tool to evaluate line integrals of analytic functions over closed curves. Cauchy's Residue Theorem can be used to evaluate various types of integrals of real valued functions of real variable. In this paper we present a new technological approach to solve line integrals of analytical functions using MATLAB.

Keywords

Cauchy's Residue Theorem, MATLAB (version 7.14.0.739).

I. Introduction

Many important definite integrals can be evaluated by applying the Residue Theorem [1-2] to properly chosen integrals. The contour chosen will consist of straight lines and circular arcs. MATLAB is integrated computer software which has three functions: Symbolic computing numerical computing and graphics drawing. MATLAB [3-4] is capable to carry out many functions including computing polynomials and rational polynomials, solving equations and computing many kind of mathematical expressions. One can also use MATLAB to calculate the limit, derivative, integral and Taylor series of some mathematical expressions. With MATLAB, The graphs of functions with one or two variables can be easily drawn in selected domain.

II. Cauchy's Residue Theorem

If F(z) is analytic in closed curve C except at a finite number of singular points within C, then

$$\int_C F(z)dz = 2\pi i(s)$$

Where s = sum of residues at the singular points within C.

Residues: The coefficient of $(z-a)^{-1}$ in the expansion of F(z) around an isolated singularity is called the residue of F(z) at that point.

Calculation of Residues:

(i) If F(z) has a simple pole at z=a then

$$resF(z) = \lim_{z\to a}(z-a)F(z)$$

(ii) If F(z) has a pole of order n at z=a then

$$resF(z) = \frac{1}{(n-1)!}\lim_{z\to a}\left[\frac{d^{n-1}}{dz^{n-1}}(z-a)^n F(z)\right]$$

III. General Method to solve integral by Cauchy's Residue Theorem

Example 1: Evaluate $\int_C \frac{z-3}{z^2+2z+5}dz$ where C is the circle $|z+1-i| = 2$

Solution: The poles of $F(z) = \frac{z-3}{z^2+2z+5}$ are given by $z^2+2z+5=0$ i.e. $-1\pm 2i$.

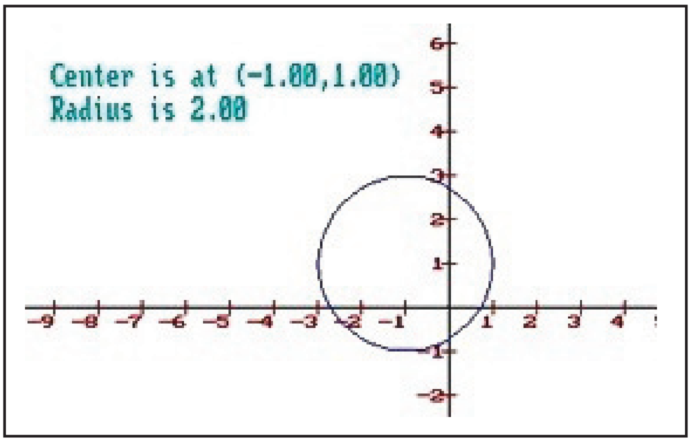


Fig. 1: Circle of $|z+1-i| = 2$

Here only one pole z=-1+2i (Fig.1) lies inside the circle C: $|z+1-i| = 2$. Therefore F(z) is analytic within C except at this pole.

$$resF(-1+2i)$$

$$= \lim_{z\to -1+2i}\left[(z-(-1+2i))\frac{z-3}{z^2+2z+5}\right]$$

$$= i+\frac{1}{2}$$

Hence by Cauchy's Residue Theorem

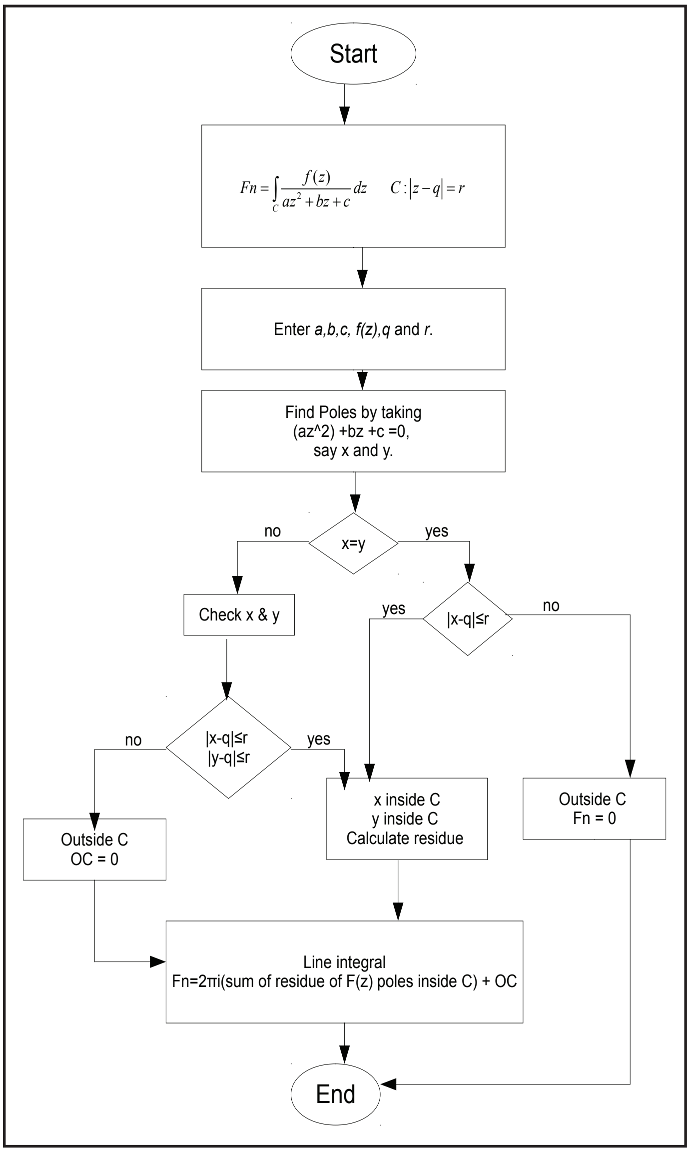
$$\int_c F(z)dz = 2\pi i\left(i+\frac{1}{2}\right) = \pi(-2+i)$$

IV. MATLAB implementation to Solve Integral by Cauchy Residue Theorem

MATLAB has some tools for solving quadratic equation and function of differentiation. We have used these functions to solve above integral.

V. MATLAB Algorithm for the integral of the form:

$$Fn = \int_C \frac{f(z)}{az^2+bz+c}dz \qquad C:|z-q| = r$$

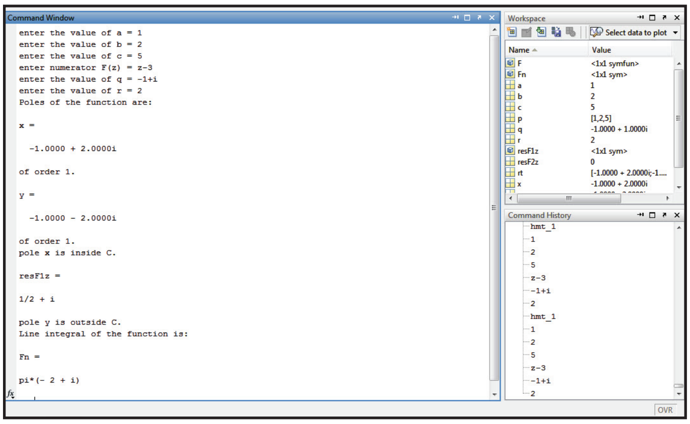


VI. Output of MATLAB Programming

Example 1: Evaluate

$$\int_C \frac{z-3}{z^2+2z+5}dz \text{ ; where C is the circle } |z+1-i|=2$$

MATLAB Output:



enter the value of a = 1
enter the value of b = 2
enter the value of c = 5
enter numerator F(z) = z-3
enter the value of q = -1+i
enter the value of r = 2
Poles of the function are:

x = -1.0000 + 2.0000i of order 1.
y = -1.0000 - 2.0000i of order 1.

pole x is inside C.

resF1z = 1/2 + i

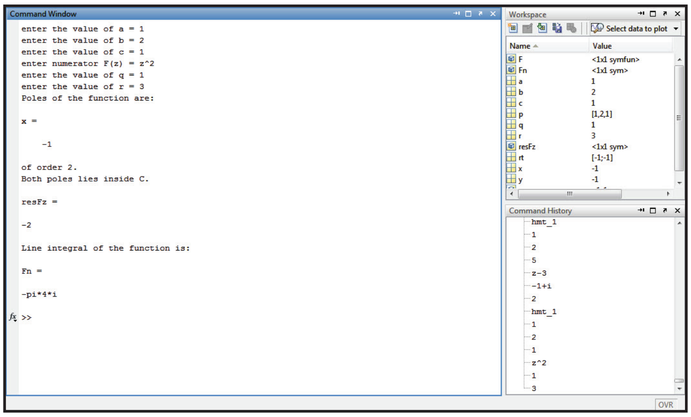
pole y is outside C.

Line integral of the function is:
Fn = pi*(- 2 + i)

Example 2: Evaluate

$$\int_C \frac{z^2}{z^2+2z+1}dz \qquad C:|z-1|=3$$

MATLAB Output:



enter the value of a = 1
enter the value of b = 2
enter the value of c = 1
enter numerator F(z) = z^2
enter the value of q = 1
enter the value of r = 3
Poles of the function are:

x = -1 of order 2.
Both poles lies inside C.

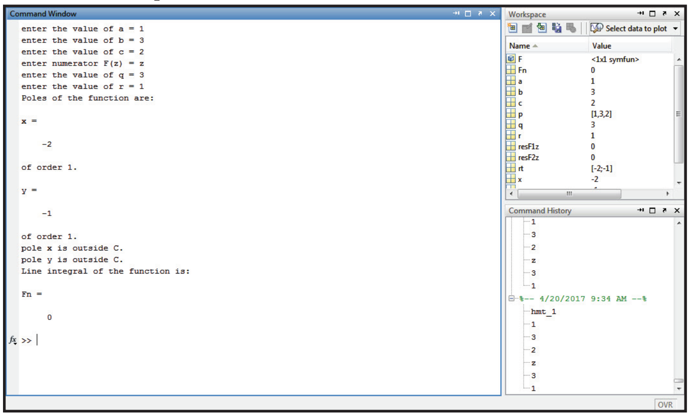
resFz = -2

Line integral of the function is:
Fn = -pi*4*i

Example 3: Evaluate

$$\int_C \frac{z}{z^2+3z+2}dz \qquad C:|z-3|=1$$

MATLAB Output:



enter the value of a = 1
enter the value of b = 3
enter the value of c = 2
enter numerator F(z) = z
enter the value of q = 3
enter the value of r = 1

Poles of the function are:

x = -2 of order 1.
y = -1 of order 1.

pole x is outside C.
pole y is outside C.

Line integral of the function is:
Fn = 0

VI. Conclusion

Mathematical software saves a lot of time in routine Cauchy's Residue Theorem is used to solve line integral of analytical function. This paper presents examples of MATLAB applications in mathematical computation. From these examples, we can see MATLAB can be a significant tool in mathematical computation and graphical representation of the functions.

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